Maximum Velocity Analysis of Parallel Manipulators

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Abstract

In order to analyse the maximum velocity of parallel manipulators in operational space, a graphical method based on the analysis of the algebraic inequalities describing the constraints on the kinematics model is presented. We consider a general 6 d.o.f parallel manipulator and assume that all articular velocities are bounded by the same limit. The method is then applied to the Conventional Stewart Platform chosen as a reference and a New Parallel Manipulator for their performance comparisons. For these two parallel manipulators, respective formula of Jacobian matrix is also given.

1 Introduction

During the design of a parallel robot, it is interesting to determine at every configuration point belonging to its workspace, which the maximum velocity is allowed to move the mobile platform in a given direction of movement, and for a given bounded limit of articular velocities. This type of analysis is classified as the direct analysis problem, which is important for robot position control and trajectory generation, because the manipulator task will be optimized. As an example, for a task of transfer between two points, it's better to choose two points on the same trajectory where the velocity is maximum. There exist several methods used for the velocity analysis of spatial mechanisms, but few when dealing with parallel robots [1,2,3,4].

In this paper, we present a graphical method for the maximum velocity analysis of parallel manipulators in operational space. It is essentially based on the analysis of the algebraic inequalities describing the constraints on the kinematics model. In this method, we consider a general 6 d.o.f parallel manipulator and assume that all articular velocities are bounded. This method is similar to those developed by Yoshikawa [5], who used the hyper-ellipsoid to characterize the performance of a manipulator in terms of manipulability. Here, we use the intersection of hyperplanes to characterize the performance of a parallel manipulator in terms of maximum velocity. The procedure is applicable to the Conventional Stewart Platform (CSP) [6], which has been well known as a typical one of the parallel mechanism and to a New Parallel Manipulator (NPM) with fixed linear actuator developed at Mechanical Engineering Laboratory (MEL) in Tsukuba, Japan [7]. Respective formula of Jacobian matrix is also given in these applications. Finally, some graphical results allow to show their performance comparisons in terms of permitted zones of maximum velocity.

2 Determination of maximum velocity zones

2.1 Formulation

The differential relationship of a general manipulator [8,9] can be expressed as

\[ J \dot{x} = J_1 \dot{I} \]

where \( x \) is the end effector velocity vector and \( \dot{I} \) is the articular velocity vector. A single Jacobian matrix of general parallel manipulators can be defined as \( J = J_1^{-1} J_x \), provided \( J_1 \) is invertible. Eq. (1) yields

\[ \dot{I} = J \dot{x} \]

Note that the Jacobian \( J \) that is introduced here, and more generally for parallel robots, is the classical inverse Jacobian of serial robots. This explains why we have here \( J \) and not \( J^{-1} \) as is the case for serial robots.

A general 6 d.o.f parallel manipulator is composed of 2 rigid bodies connected by 6 actuated legs. The stationary body is referred to as the base and the moving body is referred to as the mobile platform. We choose the limit of articular velocities \( v_i \), define the Jacobian \( J \) and the velocity \( \dot{x} \) of the mobile platform as follows:

\[ |v_i| \leq \rho \quad \forall i \in \{1,6\}, \quad J = \begin{bmatrix} c_{i1} \\ c_{i2} \\ c_{i3} \end{bmatrix}_{i=1,6} \quad \text{and} \quad \dot{x} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \]

where \( \rho \) is the bounded limit of all articular velocities, \( c_{ij} \) is the \( i \)th and \( j \)th element of the Jacobian matrix \( J \), \( (v_x, v_y, v_z) \) are the three translational components and \( (\omega_x, \omega_y, \omega_z) \) three rotational components of the operational velocity. At every configuration point \( x_c \) of robot, we try to find in the hyperplane the frontier limiting possible velocities of the mobile platform. From Eq. (2), every point of the permitted zone must satisfy the following constraints:

\[ -\rho \leq c_{1i} v_x + c_{2i} v_y + c_{3i} v_z + c_{4i} \omega_x + c_{5i} \omega_y + c_{6i} \omega_z \leq \rho \quad \forall i \in \{1,6\} \]

where \( (x_c, y_c, z_c) \) are the three translational components of the position vector of the mobile platform and \( (\psi, \theta, \phi) \) three
Euler angles defining the orientation of the mobile platform with respect to the fixed base, when four components of the velocity are fixed, we can represent in a plane with different combinations the frontier limiting two remaining components of velocity of the mobile platform. As an example, supposing \( v_z = 0 \) and \( \omega_2 = \omega_3 = \omega_4 = 0 \), this yields
\[
-p \leq c_{11} v_x + c_{12} v_y \leq p, \quad \forall i \in \{1, 6\}
\]
Fig. 1 shows in the \((v_x, v_y)\) plane, lines defining the constraint equations (4) and the permitted polygonal zone.

From this method of construction, we can also build the volume of permitted velocity by drawing successive cuts for different values of a third component of operational velocity. For example, if \( \vec{v} = [v_x, v_y, 0, 0, 0]^T \), at every configuration point \( \vec{x}_c \) of robot we will have
\[
-p \leq c_{11} v_x + c_{12} v_y \leq p, \quad \forall i \in \{1, 6\}
\]
This type of intersection in 3D may result in either an empty set, or a point, or a segment, or finally a polygon or a convex polyhedron.

**2.2 Algorithm development**

Considering in case of 2D, \( 2n \) constraint inequalities of system (4) as follows.
\[
-c_i \leq a_i x + b_i y \leq c_i, \quad \forall i = 1, n
\]
The system (6) can also be rewritten in the following form.
\[
a_i x + b_i y + c_i = 0, \quad \forall i = 1, n
\]
Each equation \( a_i x + b_i y + c_i = 0 \) defines a line which divides the plan \((x,y)\) into two half planes \( P_{i1} \) and \( P_{i2} \) (respectively \( P_{i1}^* \) and \( P_{i2}^* \)).

\( P_{i1}^* \) is defined by \( a_i x + b_i y + c_i > 0 \)
\( P_{i2}^* \) is defined by \( a_i x + b_i y + c_i < 0 \)
\( P_{i1} \) is defined by \( a_i x + b_i y - c_i > 0 \)
\( P_{i2} \) is defined by \( a_i x + b_i y - c_i < 0 \)

The delimited zone, which satisfies the above constraint inequalities (7), is determined as in the following steps.

**STEP 1**: Determine all intersection points of line equations \( a_i x + b_i y + c_i = 0 \) and \( a_i x + b_i y - c_i = 0 \) for \( i = 1, n \).

**STEP 2**: Order these intersection points on each line, by sorting \( x \)-axis values of points in ascending order followed by corresponding \( y \)-axis values. These obtained intersection points cut the lines out of segments.

**STEP 3**: For each segment, determine its middle point. Calculate the normal of the segment at this middle point. Choose two opposite points on this normal with very small distance from middle point and calculate the values of inequalities for these two points. For each point, if these values of inequalities are all satisfied, then the considered segment belongs to the frontier of the delimited zone.

**STEP 4**: Redo the **STEP 3** for all segments of each line and then for all lines.

**STEP 5**: After determining all segments which are solutions of the constraint inequalities (7), the delimited zone is finally obtained by connecting successively these segments.

The delimited zone in 2D is a convex polygon composed of \( N \) segments \((N \leq 2n)\). The solution \( V_{max} = \{ \vec{x} \}_{max} \) corresponds to the summit of the polygon farthest from the origin of the velocity space, and the norm of the vector from the origin to this summit represents \( V_{max} \). The direction of maximum velocity is the tangent to the trajectory of maximum velocity.

**2.3 Surface and volume computation**

The velocity surface of the polygon is composed of \( N \) surfaces of triangles. The three summit points of a triangle are the origin of the velocity space and the two successive points defining a segment of the polygon. The surface of the polygon is computed as follows
\[
S_{\text{polygon}} = \sum_{i=1}^{N} S_{\Delta_i}, \quad \text{where } S_{\Delta_i} \text{ represents the surface of a triangle } \Delta_i \text{ illustrated in Fig. 2.}
\]

\[
S_{\Delta_i} = \frac{1}{2} (e_i \cdot h_i)
\]

The volume \( \Omega_{\text{domain}} \) in case of 3D, is computed as follows.
\[
\Omega_{\text{domain}} = \sum_{v_z=0}^{v_{z_{max}}} S_{\text{polygon}(v_z)} \cdot \Delta v_z
\]

where \( S_{\text{polygon}(v_z)} \) is the surface of polygon for every third component \( v_z \) of velocity space and \( \Delta v_z \) is the increment of \( v_z \) axis.
3 Applications

The manipulators used in this example are the Conventional Stewart Platform and the New Parallel Manipulator with fixed linear actuator at MEL. For performance comparison study, the principal geometric parameters and the linear actuators of these two parallel manipulators are assumed to be identical. They differ only from their mechanism architectures. The way to choose the geometric parameters of the New Parallel Manipulator knowing those of the Conventional Stewart Platform is shown in Appendix.

3.1 Kinematic analysis of the Conventional Stewart Platform

The geometric model of the Conventional Stewart Platform and its parameters are shown in Fig. 3. \( z_i \) is a unit vector of the leg \( i \) \( (i \in [1,6]) \). \( p \) is a center location vector of the mobile platform. \( l_i \) is a scalar variable controlled by the actuator \( i \). Based on the geometric relation the following equation is derived

\[
l_i z_i = p + s_i - p_{hi}
\]

where \( s_i = \mathbf{R} \mathbf{p}_{hi} - \mathbf{p}_h \) is the position of the center of the upper ball joint connecting leg \( i \) to the mobile platform and expressed in the mobile platform frame, \( \mathbf{R} \) is a 3 by 3 orthogonal matrix representing the orientation of the mobile platform with respect to the base. \( \mathbf{p}_{hi} \) represents the position of the center of the lower ball joint connecting leg \( i \) to the base platform. Differentiating Eq. (10) gives

\[
l_i z_i + l_i z_i = V + \Omega \times s_i
\]

With \( \mathbf{p} = V \) and \( s_i = \Omega \times s_i \), where \( \times \) is the cross product of two vectors, \( \mathbf{V} = [v_x, v_y, v_z]^{T} \) is the translational component and \( \Omega = [\alpha_x, \alpha_y, \alpha_z]^{T} \) rotational component of the velocity \( \dot{X} \) of the mobile platform, Eq. (11) is written as

\[
l_i z_i + l_i z_i = V + \Omega \times s_i
\]

Multiplying on both sides of Eq. (12) by \( z_i^{T} \) yields

\[
l_i = z_i^{T} V + z_i^{T} (\Omega \times s_i) = z_i^{T} V + (\Omega \times s_i)^{T} z_i
\]

By using the following property of the cross and inner products of vectors \( (a \times b)^{T} \cdot c = (b \times c)^{T} \cdot a \), we have

\[
l_i = [z_i^{T} (s_i \times z_i)^{T}] [V \Omega]
\]

Thus, \( J_i \) is an identity matrix 6 by 6 and

\[
J_i = J_s = \begin{bmatrix} z_i^{T} & (s_i \times z_i)^{T} \end{bmatrix}_{6 \times 6}
\]

3.2 Kinematic analysis of the MEL New Parallel Manipulator

The geometric model of the New Parallel Manipulator and its parameters are shown in Fig. 3. \( a_i \), \( w_i \) and \( z_i \) are unit vectors of the linkage \( i \) \( (i \in [1,6]) \). \( c \) is a fixed length of the moving linkage. The definition of other parameters are the same as in Stewart Platform. Based on the geometric relation the following equation is derived

\[
l_i a_i + c z_i = V + s_i - p_{hi}
\]

Here \( p_{hi} \) is the position of the center of the linear actuator \( i \) fixed on the base platform.

Differentiating Eq. (16) gives

\[
l_i a_i + c z_i = V + (\Omega \times s_i)
\]

By multiplying on both sides of Eq. (17) by \( z_i^{T} \) and taking into account the previous results, it yields

\[
(z_i^{T} a_i) l_i = \begin{bmatrix} z_i^{T} (s_i \times z_i)^{T} \end{bmatrix} [V \Omega]
\]

Thus, with \( z_i^{T} a_i \neq 0 \) \( \forall i \in [1,6] \) we have

\[
J_i = \begin{bmatrix} 0 & z_i^{T} a_i & 0 \\ 0 & 0 & \ldots \end{bmatrix}_{6 \times 6}
\]

and

\[
J = \begin{bmatrix} z_i^{T} & (s_i \times z_i)^{T} \end{bmatrix}_{6 \times 6}
\]

4 Results

Figs. 4 and 5 present in 2D \( (v_x, v_y) \) and 3D \( (v_x, v_y, v_z) \) views the zones of permitted velocities of the two parallel manipulators for different positions \( x_c = [x_c, y_c, z_c] \psi \theta \phi \) and fixed rotational components \( (\alpha_x, \alpha_y, \alpha_z) \). Fig. 6 presents the performance comparisons based on maximum velocity of the two manipulators in 2D for a nominal position and in 3D for a nominal straight line trajectory in z-axis. We notice that for a given position when \( v_z \) is increasing, the corresponding surface of the intersection zone is distorted and decreasing. By using 3D representation, we can determine the maximum values of the three translational components \( (v_x, v_y, v_z) \). Figs. 4 and 5 show that the best performance of the two manipulators is obtained in the example nominal position \( x_c = [0 0 0.596m 0 0 0]^{T} \) with \( \alpha_x = \alpha_y = \alpha_z = 0 \), where the maximum ranges of \( (v_x, v_y, v_z) \) are higher and the surface respectively volume is more important. Note that this nominal position is bounded by the largest workspace of
these parallel manipulators [7]. Fig. 6 a) shows that the norms of maximum operational velocity of the two manipulators are very close. But, in this nominal position with $\omega_x = \omega_y = \omega_z = 0$ and $v_z = 0$ the polygon of the NPM is very close to a circle, while those of the CSP is exactly a hexagon. This suggests that the NPM has the most uniform and isotropic velocity zone, which can facilitate the motion control and trajectory generation from nominal position. Fig. 6 b) shows that the two manipulators have almost the same maximum velocity performance on the straight line nominal trajectory when $z_c < z_{\min}^2 + z_{\max} = 0.596m$. For $z_c \geq 0.596m$, it seems that the maximum velocities in z-axis of the CSP are higher than those of the NPM. However, the permitted velocity zones in $(v_x, v_y)$ space of the NPM are more isotropic and uniform.

5 Conclusion

In this paper, a graphical method with its algorithm for analyzing the maximum velocity of parallel manipulators in operational space has been presented. The advantage of the method based on velocity is that the constraint equations are simple by just knowing the Jacobian matrix of the manipulator at a given configuration point. The drawing results allow the designer to have a quick look of his parallel manipulator performance in terms of maximum velocities. The results show that from a certain nominal position, the CSP performance is characterized by its superior high speed, whereas the NPM performance is based on the isotropy and uniformity property, which is important for robot position control and trajectory generation.

Since some researchers [5,10,11] used the Jacobian in some way for performance evaluation of manipulators, it is also believed that this study on maximum velocity drawing can be proposed as an evaluation performance method for the kinematic design of high speed parallel manipulators. The numerical surface or the volume of the permitted velocity zone may be used as a kinematic system performance measure.

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References


Fig. 3: Geometric model of the two parallel manipulators

Nominal position: \( x_0 = y_0 = 0; z_0 = 0.596\text{m} \), \( \psi = \theta = \phi = 0^\circ \), \( \omega_1 = \omega_2 = \omega_3 = 0 \)

Fig. 4: Maximum velocity of the Conventional Stewart Platform

Nominal position: \( x_0 = y_0 = 0; z_0 = 0.596\text{m} \), \( \psi = \theta = \phi = 0^\circ \), \( \omega_1 = \omega_2 = \omega_3 = 0 \)

\( \omega_2 = 5 \text{ rad/s} \)
Fig. 5: Maximum velocity of the MEL New Parallel Manipulator

- **Nominal position**: \((x_0 = y_0 = 0; z_0 = 0.596m), (\psi - \theta - \phi = 0), (\alpha_0 - \alpha_I = \alpha_2 = 0)\)

- **Nominal trajectory**: \((x_0 = y_0 = 0; z_0 = 0.596m), (\psi - \theta - \phi = 0), (\alpha_0 - 5\text{ rad/s}; \alpha_I = \alpha_2 = 0)\)

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**Fig. 6: Performance comparisons between the two manipulators**

- **Stewart Platform**
  - **Z_{\text{max}} = 0.836**
  - **\|F_{\text{max}}\| = 5.502 m/s**
  - **Surface = 78.647 (m/s)^2**

- **MEL Parallel Manipulator**
  - **Z_{\text{max}} = 0.356**
  - **\|F_{\text{max}}\| = 5.087 m/s**
  - **Surface = 72.236 (m/s)^2**

- **a)** Nominal position: \((x_0 = y_0 = 0; z_0 = 0.596m), (\psi - \theta - \phi = 0)\) with \((\alpha_0 - \alpha_I = \alpha_2 = 0), (r = 0)\)

- **b)** Nominal trajectory: \((x_0 = y_0 = 0; z_0), (\psi - \theta - \phi = 0)\) with \((\alpha_0 - \alpha_I = \alpha_2 = 0)\)